DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 2003 – MID-TERM #2
NOVEMBER 15, 2017

NAME:____________________________________ **STUDENT I.D.:** ______________

Closed Book exam: no books, notes, calculators

1. Let $z = \ln(x + y)$, and $x = \cos(t), y = \sin(t)$. Find $\frac{dz}{dt}$.

2. Let $f(x, y, z) = xye^z$.
   (a) Find $\vec{\nabla} f$. Find $\vec{\nabla} f(1, 3, 0)$.
   (b) Let $\vec{u}$ be the unit vector $\langle 1/3, 2/3, 2/3\rangle$. Compute $D_{\vec{u}} f(1, 3, 0)$.
   (c) Find an equation for the tangent plane to the surface $xye^z = 3$ at the point $(1, 3, 0)$.

3. Consider the function $f(x, y) = 2xy - 4x + x^2 - y^2$.
   (a) Find the critical point of $f(x, y)$. Classify the critical point as a saddle point, local maximum or local minimum.
   (b) Use Lagrange multipliers to find the maximum and minimal values of the function $f(x, y)$ subject to the constraint $g(x, y) = 2xy - y^2 - 2x^2 + 2x = 0$.
   (c) The inequality $g(x, y) = 2xy - y^2 - 2x^2 + 2x \geq 0$ defines a closed bounded region. Combine parts (a) and (b) to find the absolute maximum and minimum of $f(x, y)$ in this region, and where they occur.

4. Evaluate
$$\int_0^2 \int_0^1 (x + e^{-y})\, dx\, dy.$$

5. Evaluate
$$\int_0^1 \int_y^1 (2x + y)\, dx\, dy.$$