Solutions

DEPARTMENT OF MATHEMATICS & STATISTICS

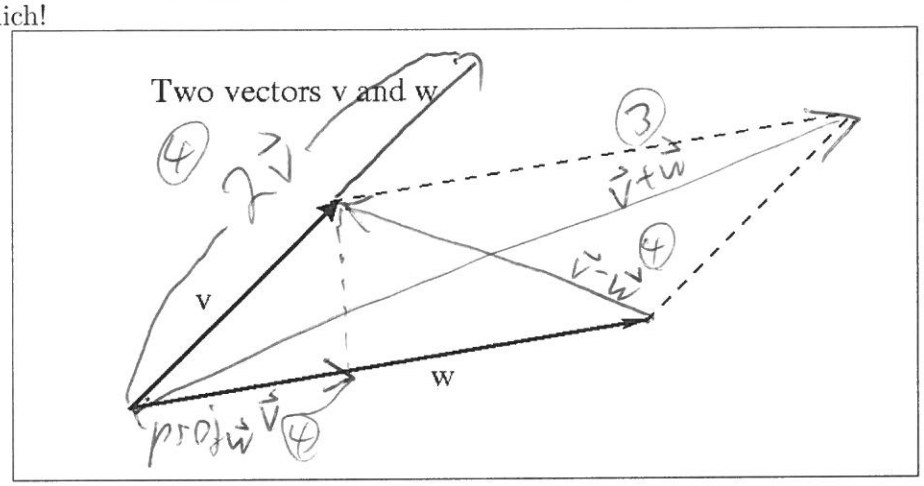
MATH 2003 - MID-TERM #1

OCTOBER 17, 2017

NAME:_____ **STUDENT I.D.:** _____

Closed Book exam: no books, notes, calculators

[15] 1. The sketch below shows two vectors $\vec{v}$ and $\vec{w}$. On the sketch clearly show and label $\vec{v}-\vec{w}$, $2\vec{v}$ and $\vec{v}+\vec{w}$ and $\text{proj}_{\vec{w}}\vec{v}$. (The projection of $\vec{v}$ onto $\vec{w}$). Be sure to indicate which vector is which!



2. Let $\vec{v} = \langle 1, 0, -1\rangle$ and $\vec{w} = \langle 3, -2, 1\rangle$.

[25]
(5) (a) Compute $\vec{v}\cdot\vec{w}$ $= 2$

(8) (b) Compute $\vec{v}\times\vec{w}$ $= \langle -2, -4, -2\rangle$

(6) (c) Compute the cosine of the angle between $\vec{v}$ and $\vec{w}$. $\cos\theta = 1/\sqrt{7}$

(6) (d) Compute $\text{proj}_{\vec{w}}\vec{v}$. $= \langle \frac{3}{7}, -\frac{2}{7}, \frac{1}{7}\rangle$

3. Consider the three points $A(1,0,1)$, $B(1,-2,0)$, $C(2,1,2)$.

[20]
(a) Find a normal vector for the plane containing the three points.

(b) Find the equation of the plane that passes through the points A, B, and C, either in linear or vector form.

4. Consider the line given by vector form $\vec{x} = \langle 1+2t, 3-2t, 5+4t\rangle$ and they plane given by
[10] the linear equation $x - y - z = 3$. Are they parallel, perpendicular or neither?

[20] 5. (a) Find the partial derivatives $\partial f/\partial x, \partial f/\partial y$ of the function $f(x,y) = x\sin(x+y)$.

(b) Find the linear equation for the tangent plane to the graph of $z = f(x,y)$ at the point $(-1,1,0)$.

[10] 6. Draw at least 5 level curves of the function $z = x^2 - y^2$.

3. $\vec{AB} = \langle 0, -2, -1 \rangle$ (4)

$\vec{AC} = \langle 1, 1, 1 \rangle$ (4)

$\vec{BC} = \langle 1, 3, 2 \rangle$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -2 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \langle -1, -1, 2 \rangle$$ (8)

$$\boxed{-x - y + 2z = 1}$$ (4)

4. $\langle 2, -2, 4 \rangle$ (2) $\cdot \langle 1, -1, -1 \rangle$ (2) $= 0$ (3)

So **parallel** (3)

5. $\frac{\partial f}{\partial x} = \sin(x+y) + x\cos(x+y)$ (4)

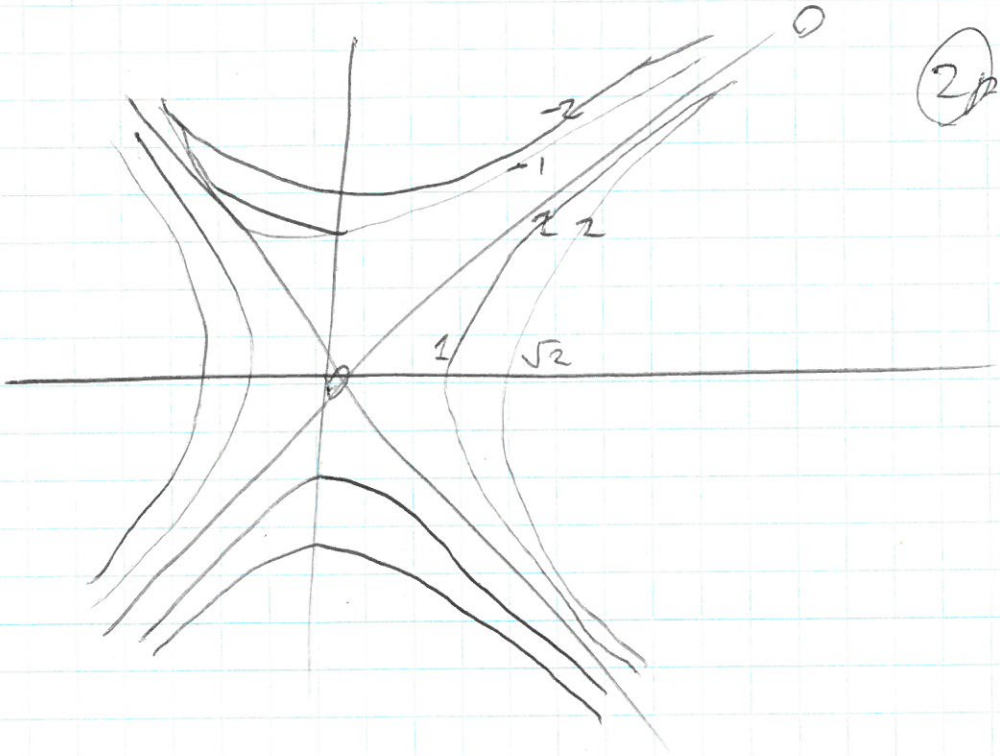
$\frac{\partial f}{\partial y} = x\cos(x+y)$ (4)

$\frac{\partial f}{\partial x}(-1,1) = -1$ (4) $\quad \frac{\partial f}{\partial y}(-1,1) = -1$ (4)

$z - 0 = -1(x - (-1)) - 1(y - 1)$ (4)

$z = -x - y$

5.



(2 pts/curve.