2003 Midterm #2 Solutions

1. (10) $\frac{\partial z}{\partial x} = \frac{1}{x+y}$ $\frac{\partial z}{\partial y} = \frac{1}{x+y}$ $\frac{dx}{dt} = -\sin t$ $\frac{dy}{dx} = \cos t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dz}{dt} = \frac{-\sin t}{x+y} + \frac{\cos t}{x+y}$$

2. (8) a) $\vec{\nabla} f = \langle ye^z, xe^z, xye^z \rangle$
$\vec{\nabla} f(1,3,0) = \langle 3, 1, 3 \rangle$

(6) b) $\langle 3,1,3 \rangle \cdot \langle 1/3, 2/3, 2/3 \rangle = 11/3$

(6) c) $\langle 3,1,3 \rangle \cdot (\langle x,y,z \rangle - \langle 1,3,0 \rangle) = 0$
$3x + y + 3z = 6$

3. (10) a) $\frac{\partial f}{\partial x} = 2y - 4 + 2x$, $\frac{\partial f}{\partial y} = 2x - 2y$
$=0$ $=0$

$\rightarrow x = y$, $x = 1 \rightarrow (1,1)$ $D = \begin{vmatrix} 2 & 2 \\ 2 & -2 \end{vmatrix} = -8$
saddle.

b) $2y - 4 + 2x = \lambda(2y - 4x + 2)$
$2x - 2y = \lambda(2x - 2y)$
$2xy - y^2 - 2x^2 + 2x = 0$

$\rightarrow x = y \rightarrow 2x - x^2 = 0 \rightarrow x = 0, 2$
$\rightarrow \lambda = 1 \rightarrow x = 1 \rightarrow 2y - y^2 = 0 \rightarrow y = 0, 2$

$(0,0)$, $(2,2)$	$(1,2)$, $(1,0)$
$f\downarrow$ $f\downarrow$	$\downarrow f$ $\downarrow f$
0 0	-3 -3
max	min

(4) c) $f(1,1) = -2$
max of 0 at $(0,0)$ $(2,2)$
min of -3 at $(1,2)$, $(1,0)$

(10) 4. $\int_0^2 \int_0^1 (x + e^{-y})\, dx\, dy = \int_0^2 \left(\frac{x^2}{2} + xe^{-y}\right)\Big]_0^1 dy = \int_0^2 \left(\frac{1}{2} + e^{-y}\right) dy = \left(\frac{1}{2}y - e^{-y}\right)\Big]_0^2 = 2 - e^{-2}$

5. $\int_0^1 (x^2 + xy)\Big|_{x=y}^{x=1} dx = \int_0^1 (1 + y - 2y^2)\, dy = 1 + \frac{1}{2} - \frac{2}{3} = 5/6$

(10)