

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2203 - MID-TERM #2

NOVEMBER 15, 2017

NAME: Solutions STUDENT I.D.: _____

[50] Closed Book exam: no books, notes, calculators

- [10] 1. (a) Use the Euclidean algorithm to find the gcd of 19 and 46. $gcd = 1$
 (b) Find integers s and t so that $19s + 46t$ equals your answer from (a). $s = 17, t = -7$

- [10] 2. Let p and q be prime numbers that are not equal.
 (a) Does $p^3q^4 | p^5q^3$? Explain your answer. No $4 \notin 3$,
 (b) Determine $gcd(p^2q^3, p^4q)$. p^2q

- [4] 3. How many distinct divisors does 7^311^4 have? $(3+1)(4+1) = 20$

- [4] 4. Prove that

$$\binom{n}{k} \binom{k}{j} = \binom{n}{j} \binom{n-j}{k-j}$$

- [4] 5. How many ways are there to rearrange the 12 letters AAABBBBCCCCC.

$$\frac{12!}{3!4!5!}$$

- [4] 6. The Math Society has 23 members. How many ways are there to choose a president, vice-president and treasurer? $23 \cdot 22 \cdot 21$

- [9] 7. (a) How many ways can 14 cans of soda be handed out to 5 people?
 (b) How many ways are there if everyone must get at least one can?

a) $\binom{18}{4}$ b) $\binom{13}{4}$

- [5] 8. Let x, y be integers. Suppose that $x + y$ is even. Prove that $x^2 - y^2$ is divisible by four.

4. $\binom{n}{k} \binom{k}{j} =$ (choose subset of size k inside set of size n)
 \times (choose j from the set of size k)

$=$ ((choose subset of size j from n) ~~not~~)

\times (choose subset of size $k-j$ from left over)

$= \binom{n}{j} \binom{n-j}{k-j}$ (Not chosen here = subset of size k above) (Not chosen from above)

□

8. Various ways to do this. One way!

$x+y$ even if $(x \text{ even } \& \ y \text{ even})$
or $(x \text{ odd } \& \ y \text{ odd})$

$\Rightarrow x-y$ even in either case.

So $x+y = 2n$, $x-y = 2m$

$$(x+y)(x-y) = x^2 - y^2 = 4nm \quad \text{so}$$

$$4 \mid x^2 - y^2$$

