Final Exam (MATH 2203 F15) Name & ID:

5 marks **1.** If $A = [0,4]$ and $B = [2,9)$ are intervals on the real number line $\mathbb{R}$, determine each of the following:

(a) $A \cap B$
(b) $A \cup B$
(c) A^c
(d) $A \setminus B$
(e) $(A \cup B) \setminus (A \cap B)$

5 marks **2.** Find a formula for the sum of the first n odd integers. Prove that the formula is true (your formula should be of the form $\sum_{i=1}^{n}(2i-1) = \ldots$).

5 marks **3.** How many times must we roll a single die in order to get the same score n times, for $n \geq 2$? (If you're not sure how to get started, answer the question for $n = 2$, then for $n = 3$.)

4 marks **4.** (a) Use the Euclidean Algorithm to find $\gcd(561, 330)$.

1 mark (b) Express the greatest common divisor as a linear combination of 561 and 330 using integer coefficients.

2 marks **5.** (a) How many arrangements are there of all the letters in SOCIOLOGICAL?

2 marks (b) In how many of these arrangements are A and G adjacent?

1 mark (c) In how many of these arrangements are all the vowels adjacent?

5 marks **6.** How many ways are there to pick a five-person basketball team from 12 possible players? How many selections include the weakest and strongest players? (Assume there is one weakest and one strongest player.)

5 marks **7.** Let R be the relation on $\mathbb{Z}$ defined by xRy if and only if $x + y$ is odd. Determine whether R is reflexive, symmetric, antisymmetric, or transitive.

8. Let $A = \{1,2,3,4\}$ and consider the poset $(\mathscr{P}(A), \subseteq)$, where $\mathscr{P}(A)$ denotes the set of all subsets of A and $\subseteq$ denotes the partial ordering on $\mathscr{P}(A)$ defined by set inclusion. Consider the following subset of $\mathscr{P}(A)$:

$$B = \{\{1,3,4\}, \{2,3\}, \{3,4\}, \{1,3\}, \{3\}\}$$

3 marks (a) Identify any maximal, maximum, minimal, minimum elements of B.

2 marks (b) Identify an upper bound and a lower bound for B. Does B have a least upper bound and/or a greatest lower bound?

9. On the set $A = \{1,2,3,11,12,13,21,22,23\}$, define a relation $\preceq$ by $m \preceq n$ if and only if the set of digits of m is a subset of the set of digits of n. For instance, $1 \preceq 11$ and $1 \preceq 21$ but $1 \not\preceq 23$.

1 mark (a) Briefly verify that $\preceq$ is a quasiorder on A (your verification need not be detailed).

2 marks (b) List the elements of each equivalence class of the induced equivalence relation.

2 marks (c) Draw the Hasse diagram for the induced partial order on the equivalence classes.

5 marks **10.** If $f : A \to B$ and C, D are subsets of B, show that $f^{-1}(C \setminus D) = f^{-1}(C) \setminus f^{-1}(D)$.