Solns Math 2013 Assgn 5

1.) $\vec{r} = \langle 2,1,3\rangle + s\langle 1,-1,1\rangle + t\langle 0,2,-3\rangle$

a) $= \langle 2+s, 1-s+2t, 3+s-3t\rangle$

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$x+3y+2z=11$

b) $x = 6\sin\varphi\cos\theta$ $\quad 0 \le \theta \le 2\pi$

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$y = 6\sin\varphi\sin\theta$ $\quad \pi/3 \le \varphi \le 2\pi/3$

$z = 6\cos\varphi$

2. $\vec{r}_u = \langle -2u, -1, 0\rangle$ $\quad \vec{r}(1,1) = (-1,-1,-1)$

$\vec{r}_v = \langle -2v, 0, -1\rangle$ $\quad u_0 = 1,\ v_0 = 1$

$\vec{r}_u \times \vec{r}_v = \langle 1, -2u, -2v\rangle$ $\quad (\vec{r}_u \times \vec{r}_v)(1,1) = \langle 1, +2, +2\rangle$

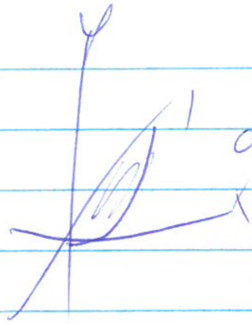
$\langle 1,-2,-2\rangle \cdot \langle x,y,z\rangle = \langle 1,-2,-2\rangle \cdot \langle -1,-1,-1\rangle$

6

$x - 2y - 2z = 3$

3.

6



$\vec{r} = \langle x, y, \sqrt{x^2+y^2} \rangle$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} i & j & k \\ 1 & 0 & x/\sqrt{x^2+y^2} \\ 0 & 1 & y/\sqrt{x^2+y^2} \end{vmatrix}$$

$$= \left\langle \frac{-x}{\sqrt{x^2+y^2}}, \frac{-y}{\sqrt{x^2+y^2}}, 1 \right\rangle$$

$$\int_0^1 \int_{x^2}^{x} \sqrt{1 + \frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}}\, dy\, dx = \int_0^1 \int_{x^2}^{x} \sqrt{2}\, dy\, dx$$

$$= \sqrt{2} \int_0^1 (x - x^2)\, dx = \sqrt{2}\left(\tfrac{1}{2} - \tfrac{1}{3}\right) = \sqrt{2}/6$$

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4. $\vec{r}(\theta,\varphi) = \langle (a+b\cos\theta)\cos\varphi,\ b\sin\theta,\ (a+b\cos\theta)\sin\varphi \rangle$

$\vec{r}_\theta = \langle -b\sin\theta\cos\varphi,\ b\cos\theta,\ b\sin\theta\sin\varphi \rangle$

$\vec{r}_\varphi = \langle -(a+b\cos\theta)\sin\varphi,\ 0,\ (a+b\cos\theta)\cos\varphi \rangle$

$$\vec{r}_\theta \times \vec{r}_\varphi = \begin{vmatrix} i & j & k \\ -b\sin\theta\cos\varphi & b\cos\theta & b\sin\theta\sin\varphi \\ -(a+b\cos\theta)\sin\varphi & 0 & (a+b\cos\theta)\cos\varphi \end{vmatrix}$$

$= \langle b(a+b\cos\theta)\cos\theta\cos\varphi,\ b(a+b\cos\theta)\sin\theta,\ b(a+b\cos\theta)\cos\theta\sin\varphi \rangle$

$|\vec{r}_\theta \times \vec{r}_\varphi| = b(a+b\cos\theta)$

$$\iint_S dS = \int_0^{2\pi}\int_0^{2\pi} (ab + b^2\cos\theta)\, d\varphi\, d\theta = 4\pi^2 ab$$

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5. $\vec{r}(y,z) = \langle y+2z^2, y, z \rangle$

$\vec{r}_y = \langle 1, 1, 0 \rangle \quad \vec{r}_z = \langle 4z, 0, 1 \rangle$

$$\vec{r}_y \times \vec{r}_z = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 4z & 0 & 1 \end{vmatrix} = \langle 1, -1, -4z \rangle$$

$|\vec{r}_y \times \vec{r}_z| = \sqrt{2+16z^2}$

$$\int_0^1\int_0^1 z\sqrt{2+16z^2}\,dy\,dz = \frac{2}{3}(2+16z^2)^{3/2}\frac{1}{32}\Big]_0^1$$

$$= \frac{1}{48}(18)^{3/2} - \frac{1}{48}(2^{3/2}) = \frac{3}{8}\cdot 3\sqrt{2} - \frac{1}{24}\sqrt{2} = \frac{26}{24}\sqrt{2}$$

$$= \frac{13}{12}\sqrt{2}$$