## NON-COMMUTATIVE GEOMETRY AND THE LOCAL LANGLANDS CONJECTURE

**Paul Baum (PSU)**

Let $G$ be a reductive p-adic group. Examples are $GL(n, F), SL(n, F)$ , etc where n can be any positive integer and $F$ can be any finite extension of the field $Q_p$ of p-adic numbers. The smooth dual of $G$ is the set of (equivalence classes of) smooth representations of $G$. The representations are on vector spaces over the complex numbers. In a canonical way, the smooth dual of $G$ is the disjoint union of countably many subsets known as the Bernstein components. Results from non-commutative geometry — e.g. Baum-Connes conjecture, periodic cyclic homology of the Hecke algebra of $G$ — indicate that a very simple geometric structure might be present in the smooth dual of $G$. The ABP (Aubert-Baum-Plymen) conjecture makes this precise by asserting that each Bernstein component in the smooth dual of $G$ is a complex affine variety. These varieties are explicitly identified as certain extended quotients. For split $G$, the ABP conjecture has recently been proved for any Bernstein component in the principal series. A corollary is that the local Langlands conjecture is valid throughout the principal series. The above is joint work with Anne-Marie Aubert, Roger Plymen, and Maarten Solleveld.

## EXACT CROSSED-PRODUCTS : COUNTER-EXAMPLE REVISITED

**Paul Baum (PSU)***

An expander or expander family is a sequence of finite graphs $X_1, X_2, X_3$, . . . , which is efficiently connected. A discrete group $G$ which"contains" an expander in its Cayley graph is a counter-example to the Baum-Connes (BC) conjecture with coefficients. Some care must be taken with the definition of "contains". M. Gromov outlined a method for constructing such a group. G. Arjantseva and T. Delzant completed the construction. The group so obtained is known as the Gromov group and is the only known example of a non-exact group. The left side of BC with coefficients "sees" any group as if the group were exact. This talk will indicate how to make a change in the right side of BC with coefficients so that the right side also "sees" any group as if the group were exact. This corrected form of BC with coefficients uses E. Kirchberg's construction of the unique minimal exact crossed-product. For exact groups (i.e. all groups except the Gromov group) there is no change in BC with coefficients

**Scalar curvature and the Gauss-Bonnet theorem for noncommutative tori**

**Farzad Fathizadeh (University of Western Ontario)**

In this talk, I will first explain the computation of the scalar curvature for the noncommutative 2-torus $A_\theta$ equipped with a general translation invariant conformal structure and a Weyl conformal factor. The metric information is encoded in the Dirac operator $D$ of a spectral triple $(A_\theta, \mathcal{H}, D)$. The local expression for the curvature is computed by finding the values $\zeta_a(0)$ of the analytic continuation of the spectral zeta functions

$$\zeta_a(s) := \mathrm{Trace}(a|D|^{-2s}), \qquad a \in A_\theta^\infty, \qquad \Re(s) >> 0.$$

Our final formula for the curvature coincides precisely with the one obtained independently by Connes and Moscovici. Then, it will be explained how the scalar curvature fits into our earlier work which generalizes the Gauss-Bonnet theorem of Connes and Tretkoff to general conformal structures on $A_\theta$. At the end, I will report on our ongoing work on extensions of the Gauss-Bonnet theorem to and computing the geometric invariants of noncommutative 4-tori endowed with their canonical complex structure. The computations in dimension 4 are more complicated since conformal perturbations of the metric violate the Kähler condition. This is joint work with Masoud Khalkhali.

**Weyl's law and Connes' trace theorem for noncommutative 2-tori**

**Farzad Fathizadeh (University of Western Ontario)***

I will consider the noncommutative two torus $A_\theta$ equipped with a general translation invariant conformal structure and a Weyl conformal factor. Using Connes' pseudodifferential calculus, the analogue of Weyl's law for $A_\theta$ will be explained. This gives the asymptotic distribution of the eigenvalues of the perturbed Laplacian on $A_\theta$ in terms of its volume. I will then sketch the proof of the analogue of Connes' trace theorem for $A_\theta$, namely the coincidence of a noncommutative residue and the Dixmier trace on classical pseudodifferential operators of order $-2$ on $A_\theta$. This is joint work with Masoud Khalkhali.

**Holomorphic Lie groupoids and meromorphic connections**

**Marco Gualtieri (University of Toronto)**

In studying geometric constructions of groupoids, we encountered a natural family of 2-dimensional smooth groupoids associated to a Riemann surface equipped with an effective divisor. I will describe these "Stokes groupoids" and their importance in the theory of meromorphic flat connections, where they are involved in the Riemann-Hilbert correspondence. Particularly intriguing is the relation with Poincar's first approach to proving the uniformization theorem for Riemann surfaces. This is joint work with Songhao Li and Brent Pym.

**Rationality of Brauer-Severi Varities of Sklyanin Algebras**

**Colin Ingalls (UNB)**

Iskovskih's conjecture states that a conic bundle over a surface is rational if and only if the surface has a pencil of rational curves which meet the discriminant in 3 or fewer points, (with one exceptional case). We generalize Iskovskih's proof that such conic bundles are rational, to the case of projective space bundles of higher dimension. The proof involves maximal orders and toric geometry. As a corollary we show that the Brauer-Severi variety of a Sklyanin algebra is rational.

**Birational Classification of Noncommutative Surfaces**

**Colin Ingalls (UNB)***

We develop the minimal model theory for orders over surfaces which generalises the existence and uniqueness of minimal algebraic surfaces. We reduce the problem to the study of pairs $(Z, a)$ where a is in the Brauer group of $k(Z)$, which we reduce further to log surfaces.

**A transformation rule for the index of commuting operators**

**Jens Kaad**

For any finite number of commuting bounded operators on a Hilbert space one can define invertibility and Fredholmness in terms of the associated Koszul complex. The index of a commuting Fredholm tuple is then defined as the Euler-characteristic. The purpose of this talk is to prove a general transformation rule for the Fredholm index under Taylor's multivariable holomorphic functional calculus. Loosely speaking we express the index in terms of local degrees of the holomorphic symbol and the locally constant index function associated with the original commuting tuple of "coordinates". Natural examples include the action of Toeplitz operators on Bergman spaces over pseudoconvex domains. The talk is based on a joint project with Ryszard Nest.

**Spectral Zeta Functions, the Gauss-Bonnet Theorem, and Scalar Curvature for Noncommutative Tori Masoud Khalkhali (Western Ontario)**

This year marks the 101th anniversary of Weyl's law on asymptotic distribution of eigenvalues of Laplacians on a bounded domain. This result and its vast ramifications in spectral geometry can be imported to noncommutative geometry thanks to Connes' notion of spectral triples. In this talk I shall give a quick review of recent joint work with Farzad Fathizadeh and also a related work by Alain Connes and Henri Moscovici on computing the scalar curvature of the

noncommutative two torus $A_\theta = C(T^2_\theta)$ equipped with an arbitrary metric. The local expression for curvature, as an element of the noncommutative torus, is computed by evaluating the value of the (analytic continuation of the) spectral zeta function $\zeta_a(s) = \mathrm{Trace}(a|D|^{-s})$ at $s = 0$ as a linear functional in $a \in C^\infty(T^2_\theta)$.

TBA

**Twisted Spin-c ctructures**

**Eckhard Meinrenken ( University of Toronto)***

TBA

**From noncommutative complex geometry to noncommutative algebraic geometry**

**Ali Motadelro (University of Western Ontario)***

We investigate the complex geometry of the quantum projective space $\mathbb{C}P^\ell_q$. Especially we define holomorphic structures on the canonical line bundles of the quantum projective space and identify their space of holomorphic sections. This determines the quantum homogeneous coordinate ring of the quantum projective space. It is shown that the fundamental class of $\mathbb{C}P^\ell_q$ is naturally represented by a twisted positive Hochschild cocycle. Also, the main statement of Riemann-Roch theorem will be verified for the quantum projective line and plane. Finally, a connection with noncommutative algebraic geometry will be discussed. In fact, one can see that our quantum homegenous coordinate ring coincides with the M. van den Bergh and M. Artin twisted homogeneous coordinate ring associated to the line bundle $\mathcal{O}(1)$ on the projective space $\mathcal{C}P^n$.

**Noncommutative quotients, Langlands parameters and algebraic varieties**

**Roger Plymen (Southampton University, UK)**

The local Langlands conjecture relates arithmetic data, based on a local nonarchimedean field $F$, with the representation theory of reductive groups such as $GL(n, F)$. In this talk, we will show how the *extended quotient*, an idea from noncommutative geometry, is useful in this context. Langlands parameters (with an additional parameter) form complex algebraic varieties. We will describe some of these varieties explicitly, especially for $GL(n)$ and the symplectic group $Sp(2n)$.

This will be a non-technical talk.

Joint work with Anne-Marie Aubert and Paul Baum.

**Geometric structure in the representation theory of reductive $p$-adic groups**

**Roger Plymen (Southampton University, UK)***

The *extended quotient*, an idea from noncommutative geometry, allows us to discern, in many examples, a definite geometric structure in the representation theory of $p$-adic groups.

We will kick off with a very geometric example: $SL(4, \mathbb{Q}_2)$.

When the $p$-adic group has connected centre, the Kazhdan-Lusztig parameters have the structure of an extended quotient $T//W$, with $T$ a complex torus and $W$ a Weyl group. We will outline a proof.

This will be a non-technical talk.

Joint work with Anne-Marie Aubert, Paul Baum and Maarten Solleveld.

**Poisson structures on Fano varieties and their quantizations**

**Brent Pym (University of Toronto)**

The degeneracy loci of a holomorphic Poisson structure are the subvarieties on which the rank of the Poisson tensor drops. I will describe work with Marco Gualtieri, in which we explain that a Poisson structure has natural "residues" along its degeneracy loci. As applications, we place strong constraints on the singular locus of the hypersurface along which a generically symplectic Poisson structure degenerates, and give new evidence for Bondal's conjecture about the dimensions of the degeneracy loci of Poisson Fano varieties. In particular, we prove the conjecture for Fano fourfolds. I will illustrate these results with some examples of Poisson structures on projective spaces whose quantizations can be explicitly described. Among these examples are Poisson structures on the symmetric powers of the projective line for which the Schwarzenberger bundles quantize to bimodules.

**Cyclic Cohomology of Lie algebras with Coefficients**

**Bahram Rangipour (UNB)**

In this talk we define the cyclic cohomology of Lie algebras with coefficients. It is shows that many classical complexes, especially Weil algebras and their truncations, can be interpreted as the cyclic complex of Lie algebras with suitable coefficients. The new coefficients are in a one to one correspondence with the SAYD modules over the enveloping Hopf algebra of the Lie algebra in question. Finally we calculate the periodic Hopf cyclic cohomology of the the enveloping Hopf algebra with coefficients in a SAYD module as the cyclic cohomology of the Lie algebra with corresponding coefficients. At the end we define the Schwartzian Hopf algebra for any dimension and show that its cohomology is calculated by our new complex which is not a Weil algebra. This is joint work with Serkan Sütlü

## Chern classes as cyclic cohomology classes of type III étale foliation groupoids

**Bahram Rangipour (UNB)***

In this take the quantum symmetry of the type III étale foliation groupoids on $\mathbb{R}^n$ is investigated as a new Hopf algebra. We completely calculate the Hopf cyclic cohomology of this Hopf algebra as the classical Chern classes. We illustrate the theory in codimenstion 1 and 2 and show that, as it is expected, the corresponding characteristic map is not injective in the level of cohomology. The kernel and image of the characteristic map is completely determined and it is shown that our result is consistent with the classical theory. This is joint work with Henri Moscovici.

## CHALENGES OF LEIBNIZ SEMINORMS

**Marc Rieffel ( UC Berkeley)**

This is a preliminary report on some investigations I am making concerning seminorms on C*-algebras that satisfy the Leibniz inequality. These seminorms are central to the concept of non-commutative (or "C*-" or "quantum") metric spaces, and to metric aspects of projective modules ("vector bundles") over them. These seminorms might possibly be of eventual interest for non-commutative algebraic geometry .

I will describe some questions that are of importance for my projects, and that are already interesting for finite-dimensional C*-algebras. I will present a thread of examples that I have found that exhibit interesting behavior, and that lead to a relation, surprising to me, with standard deviation from probability theory.

**CLASSIFICATION OF SIMPLE UNITAL REAL AT-ALGEBRAS**

**Aydin Sarraf (UNB)***

The purpose of this talk is to show that simple unital real C?-algebras arising as inductive limits of sequences of real circle algebras are classifiable by certain K-theoretical and tracial data.

**Relative cyclic homology of quantum homogeneous spaces**

**Serkan Sütlü (UNB)**

By the work of Jara-Stefan, the relative cyclic homology of a Hopf-Galois extension $A(B)^H$ can be computed by a cyclic complex that depends on the Hopf algebra $H$ and $A_B = B \otimes_{B \otimes B^o} A$ as the coefficients .

On the other hand, Hopf-Galois extensions form a particular case of quotient coalgebra-Galois extensions just as the homogeneous coalgebra-Galois extensions which involves highly nontrivial noncommutative geometric examples, such as the quantum homogeneous spaces, beyond the Hopf-Galois theory.

In this talk, for a homogeneous $H/I$-extension $B \subseteq H$ we present a new (relative) cyclic complex that computes the cyclic homology of $H/B$ in terms of the coalgebra $H/I$.

**Characteristic classes of Γ-foliations**

**Serkan Sütlü (UNB)***

We discuss the transfer of characteristic classes of $\Gamma$-foliations to the cyclic cohomology classes of the action groupoid algebra $A_\Gamma$ under two different approaches to the characteristic classes of foliations. Both of these approaches based on the transfer of classes from a certain truncation of a Weil algebra of an appropriate Lie algebra. Our method, in turn, is based on the realization of such truncated Weil algebras as Hopf-cyclic complexes. We illustrate our procedure for the smooth foliations of codimenion 1 and 2 and we present the explicit cyclic cohomology representatives in the cyclic cohomology of the action groupoid.

**Localized Index theory for Lie groupoids**

**Xiang Tang (Washington University at St. Louis)**

We define the "localized index" of longitudinal elliptic operators on Lie groupoids associated to Lie algebroid cohomology classes. We derive a topological expression for these numbers using the algebraic index theorem for Poisson manifolds on the dual of the Lie algebroid. Underlying the definition and computation of the localized index, is an action of the Hopf algebroid of jets around the unit space, and the characteristic map it induces on Lie algebroid cohomology. This map can be globalized to differentiable groupoid cohomology, giving a definition as well as a computation of the "global index". The correspondence between the "global" and "local" index is given by the van Est map for Lie groupoids.

* Talks take place at Fredericton